B.Sc-part-I(Sub) Set Theory 02-07-2020

Cartesian product of sets.

Q.1. Define Ordered pair and Cartesian product.

Ans. Ordered Pair:→

An ordered pair consists of two elements a and b such that one of them say a is designated as first member and b is taken as second member and is written in parentheses as (a,b).

In an ordered pair (a,b), a is called the first co-ordinate or first component and b the second co-ordinate or second component.

Cartesian Product:→

If A and B be any two sets, then the set of an ordered pairs whose first member belongs to set A and second member belongs to set B is called the Cartesian product of A and B in that order.

Cartesian Product of two sets A and B is denoted by $A \times B$ and read as 'A cross B' or 'product set of A and B'.

Symbolically:

$$A \times B = \{(a,b) : a \in A \wedge b \in B\}$$

Similarly, $B \times A = \{(b,a) : b \in B \wedge a \in A\}$

It is clear that $A \times B \neq B \times A$.

Solved.

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Q.2. Show that
$A \subseteq B,\ C \subseteq D \Rightarrow (A \times C) \subseteq (B \times D)$.

Soln: Let $(x,y) \in A \times C$ be arbitrary. Then

$(x,y) \in A \times C \Rightarrow x \in A \wedge y \in C$
$\Rightarrow x \in B \wedge y \in D$ [$\because A \subseteq B, C \subseteq D$]
$\Rightarrow (x,y) \in (B \times D)$

$\therefore A \times C \subseteq B \times D$.

Q.3. Show that
$A \subseteq B \Rightarrow A \times A = (A \times B) \cap (B \times A)$

Soln: Since, $A \subseteq B$
$\therefore A \cap B = A$ ——— (1)

Let $(x,y) \in A \times A$ be arbitrary. Then

$(x,y) \in A \times A \Rightarrow x \in A \wedge y \in A$
$\Rightarrow x \in A \cap B \wedge y \in A \cap B$
$\Rightarrow (x \in A \wedge x \in B) \wedge (y \in A \wedge y \in B)$
$\Rightarrow (x \in A \wedge y \in B) \wedge (x \in B \wedge y \in A)$
$\Rightarrow (x,y) \in (A \times B) \wedge (x,y) \in (B \times A)$
$\Rightarrow (x,y) \in (A \times B) \cap (B \times A)$
$\Rightarrow A \times A \subseteq (A \times B) \cap (B \times A)$ ——— (2)

Again, Let $(a,b) \in (A \times B) \cap (B \times A)$ be arbitrary. Then
$(a,b) \in (A \times B) \cap (B \times A) \Rightarrow (a,b) \in A \times B \wedge (a,b) \in B \times A$.
$\Rightarrow (a \in A \wedge b \in B) \wedge (a \in B \wedge b \in A)$
$\Rightarrow (a \in A \wedge a \in B) \wedge (b \in A \wedge b \in B)$
$\Rightarrow a \in (A \cap B) \wedge b \in (A \cap B)$
$\Rightarrow a \in A \wedge b \in A \Rightarrow (a,b) \in A \times A$

$\therefore (A \times B) \cap (B \times A) \subseteq A \times A$ ——— (3)

Now, from (2) and (3), we have
$A \times A = (A \times B) \cap (B \times A)$

Proved.